Imaging the source in heavy-ion collisions

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Workshop on two-particle interferometry and elliptic flow at RHIC

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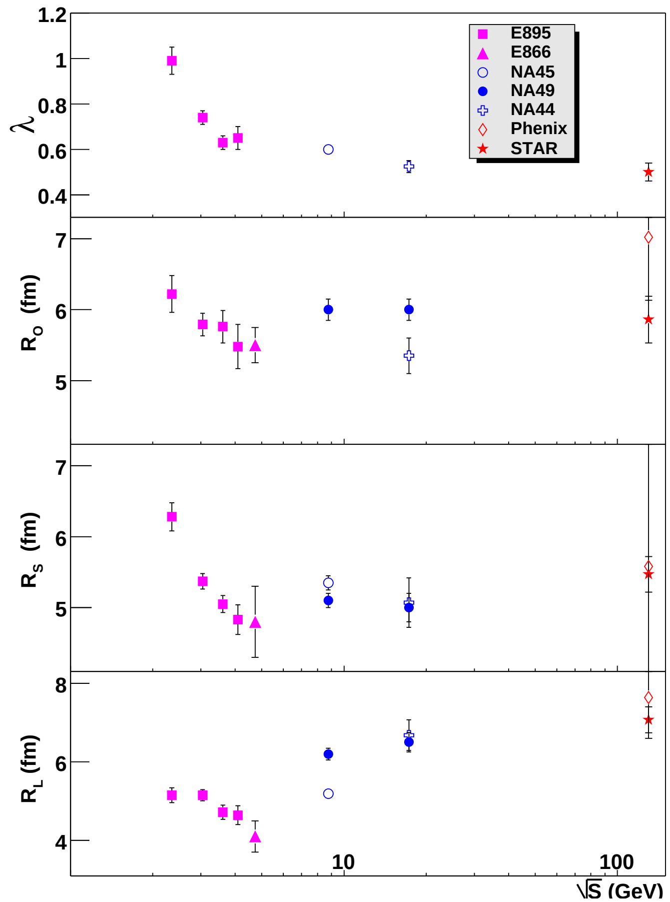
The Mystery



The Plan

1. Imaging & Interferometry
2. Another mystery: $\sqrt{s}$ dependence of proton sources?
3. $p\Lambda$ correlation suggests other ways out
4. Conclusion

Space-time information encoded in two-particle correlation function:

$$C_{\mathbf{p}}(\mathbf{q}) = \frac{dN_2/d^2\mathbf{p_1}\mathbf{p_2}}{(dN_1/d\mathbf{p_1})(dN_1/d\mathbf{p_2})}$$

- Can relate to source function via Koonin-Pratt Equation:

$$C_{\mathbf{p}}(\mathbf{q}) = \int d^3r \, |\Phi_{\mathbf{q}}(\mathbf{r})|^2 \, S(\mathbf{r})$$

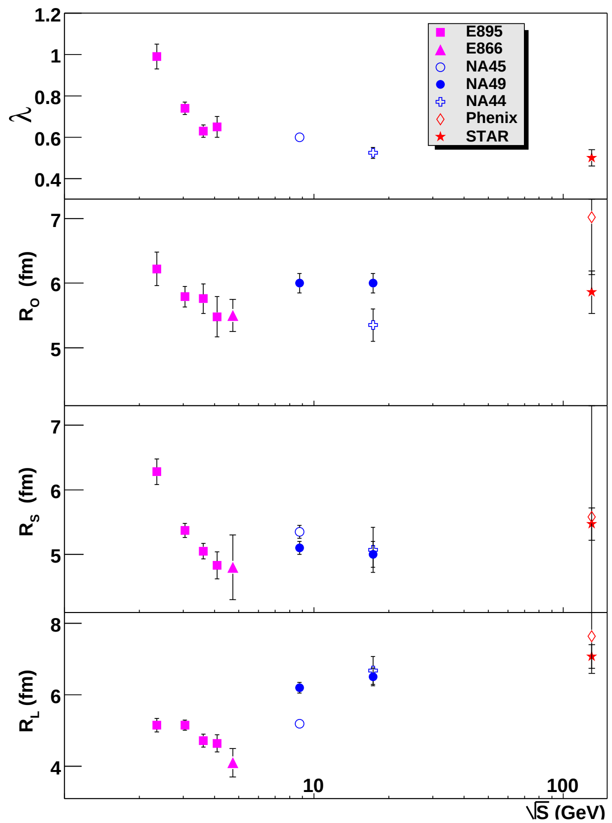
where rel. mom. is $\mathbf{q} = \frac{1}{2}(\mathbf{p_1} - \mathbf{p_2})$ and total mom. is $\mathbf{P} = \mathbf{p_1} + \mathbf{p_2}$

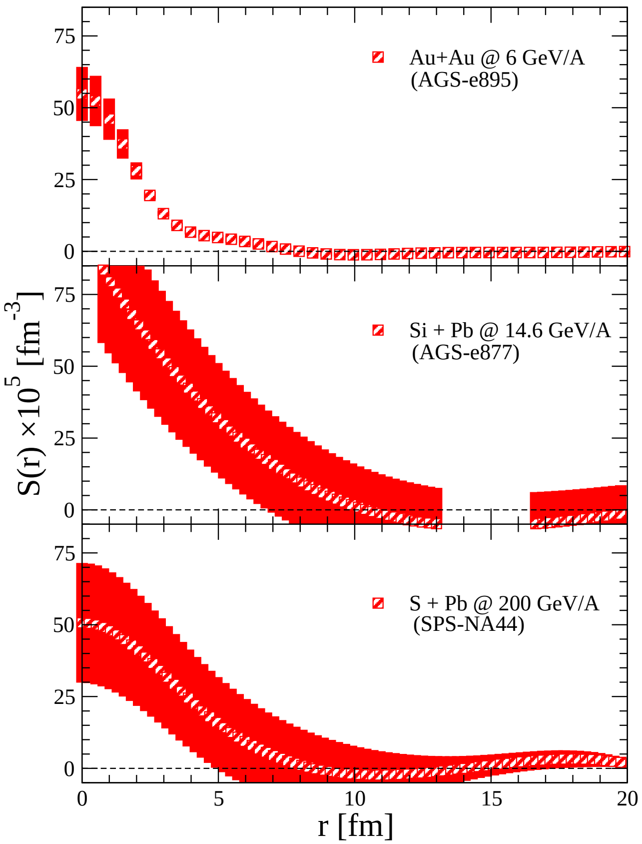
- This equation can be inverted:

1. We measured the correlation
2. We know wavefunction $\Phi_{\mathbf{q}}(\mathbf{r})$ because we can solve Schrödinger

To get at the space-time information,
we must extract the source function, $S(\mathbf{r})$

Pion and proton mysteries!





- Radii & λ parameters don't change for p's or π's !
- Same mechanism for both ?!?
- Will pΛ lets us access rest of p emission function?

The Source Function

Normalized particle emission rate (a.k.a. emission function) is

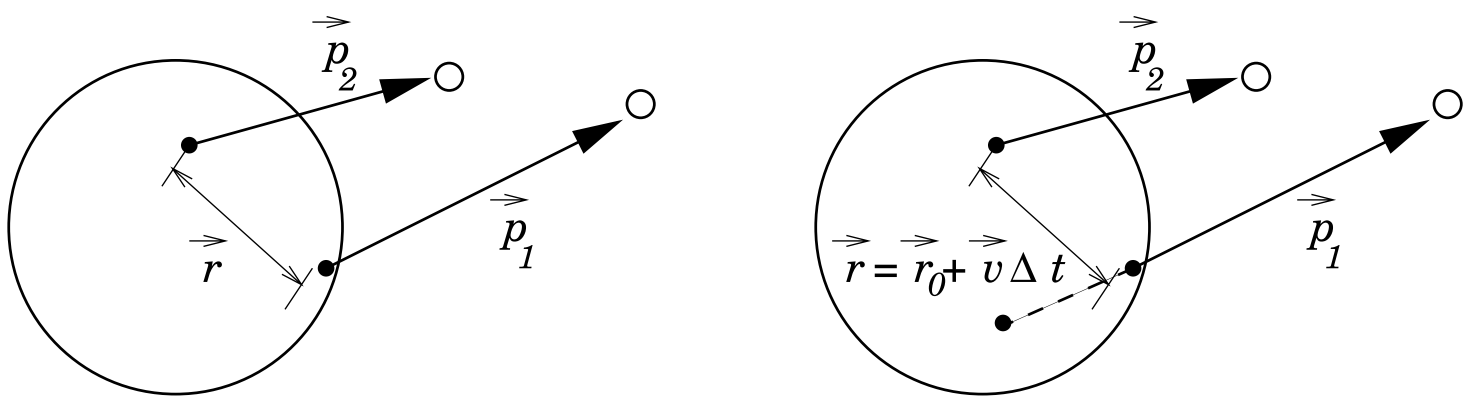
$$D(\mathbf{r},\mathbf{p},t) = \frac{1}{N_{tot}} \frac{E d^7 N}{d^3 r \, dt \, d^3 p}$$

Relative distance distribution is

$$d(\mathbf{r},\mathbf{p},t) = \int d^4 R \, D(R + r/2, \mathbf{p}) D(R - r/2, \mathbf{p})$$

and source function (what we image) is

$$S_{\mathbf{p}}(\mathbf{r}') \equiv \int dt' \, d(\mathbf{r},\mathbf{p},t)$$ in pair CM frame



Invert angle-averaged Koonin-Pratt Eq.:

$$C(q_{inv}) - 1 = 4\pi \int dr\, r^2 K_0(q_{inv}, r) S(r)$$

note: $q_{inv} = \frac{1}{2}\sqrt{-(p_1 - p_2)^2} = |\mathbf{q}|$, in pair CM

- Meson (e.g. $\pi\pi$ or KK) kernel:

$$K_0(q_{inv}, r) = \sum_{\ell} \frac{(g^{\ell}(r))^2}{(2\ell + 1)} - 1$$

to get $g_{\ell}(r)$, solve Klein-Gordon Eq. w/ Coulomb

- pp kernel:

$$K_0(q_{inv}, r) = \frac{1}{2} \sum_{js\ell\ell'} (2j + 1) \left(g_{js}^{\ell\ell'}(r)\right)^2 - 1$$

to get $g_{js}^{\ell\ell'}$, solve Schrödinger Eq. w/ Coulomb and strong pot.

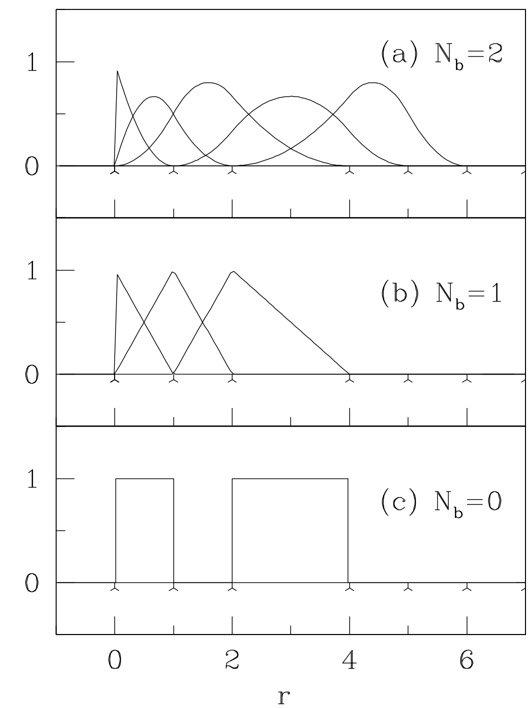
- $p\Lambda$ kernel similar to pp, but no symmetrization

We use Basis splines to encode the radial dep. of the source:

$$S(r) = \sum_j S_j B_j(r) \quad \text{with error} \quad \Delta S(r) = \sqrt{\sum_{i,j=1} \Delta^2 S_{ij} B_i(r) B_j(r)}.$$

Some Basis splines properties:

- Generalization of box and linear splines
- Continuity controlled by degree of spline
- Adjust resolution by moving knots



Ours is a linear inversion problem which may be recast into a matrix eq:

$$\mathbf{C} = \mathrm{K} \cdot \mathbf{S}$$

$\Rightarrow$ due to measurement uncertainty, problem is *ill-posed*,
i.e. no unique solution

Practical "solution" to linear inverse problem:

Best source is most probable source

- Most probable source has min χ^2. Minimization yeilds:

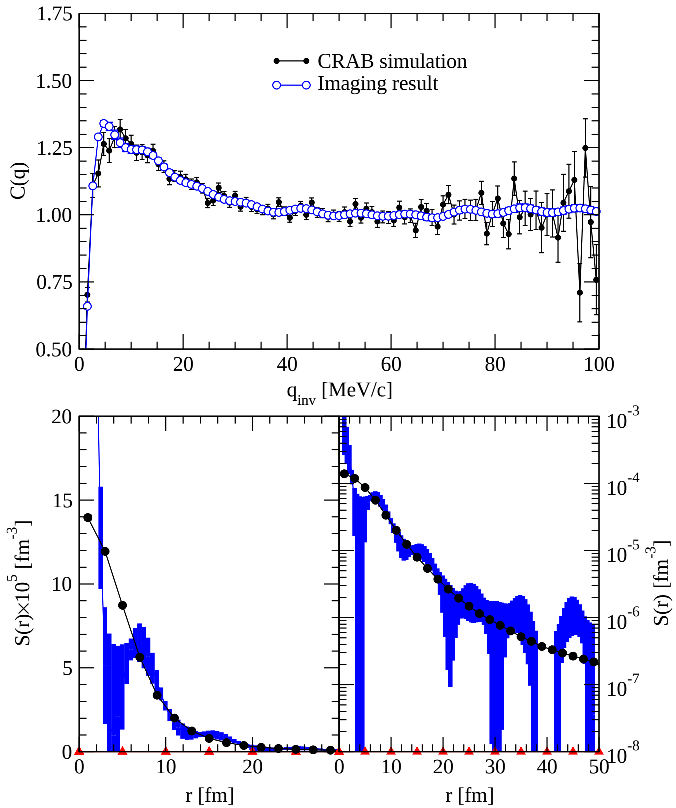
$$\mathbf{S} = \Delta^2 \mathrm{S} \cdot \mathrm{K}^{\mathrm{T}} \cdot (\Delta^2 \mathrm{C})^{-1} \cdot \mathbf{C}^{\mathbf{obs}}$$

- Covariance matrix of source is:

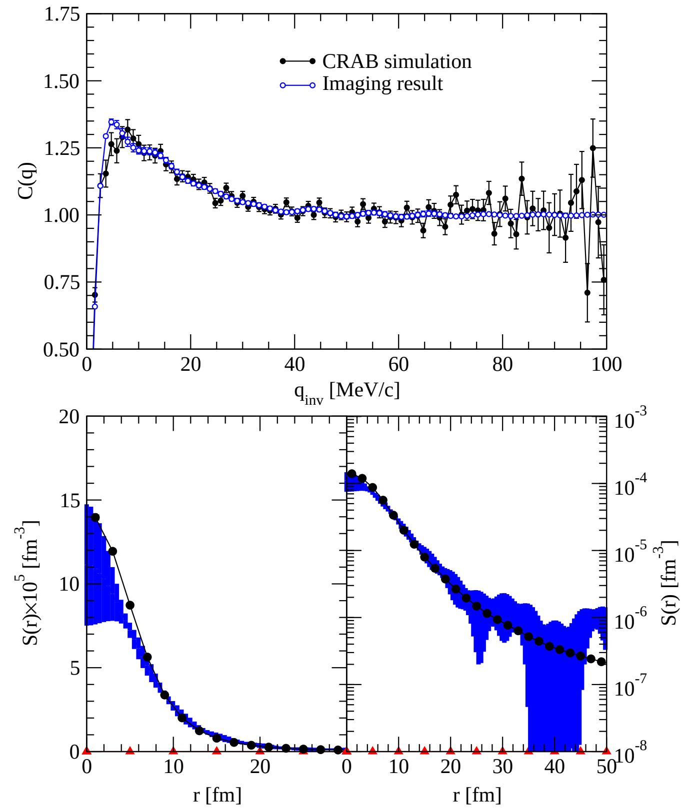
$$\Delta^2 \mathrm{S} = (\mathrm{K}^{\mathrm{T}} \cdot (\Delta^2 \mathrm{C})^{-1} \cdot \mathrm{K})^{-1}$$

$\pi^+\pi^+$ Imaging Tests in 1D

unstabilized image



constrained image

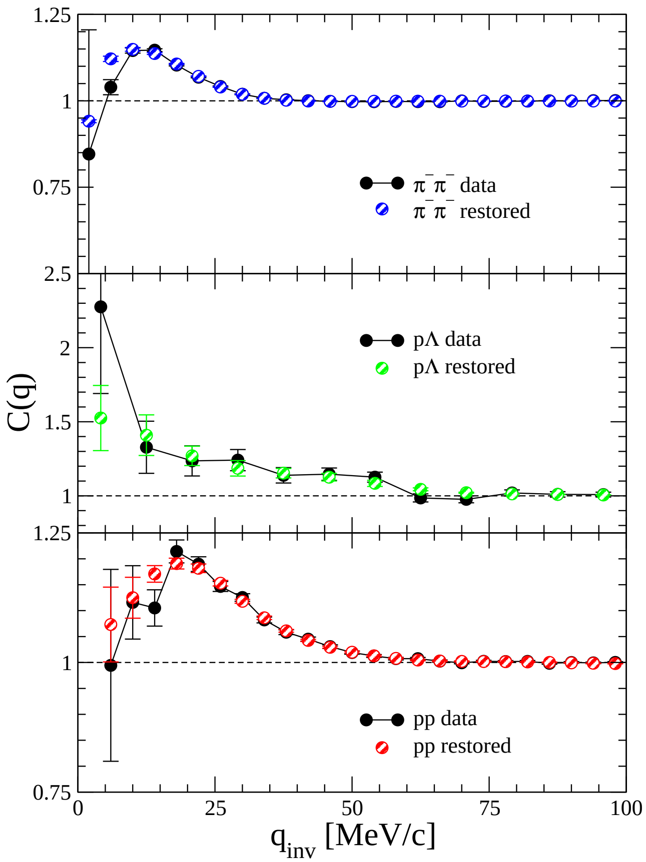


Tricks to stabilize image:

- Use constraints, e.g. source is max. at $r = 0$ fm.
- Optimize resolution by varying binning in source (not shown).

AGS-E895 CORRELATIONS

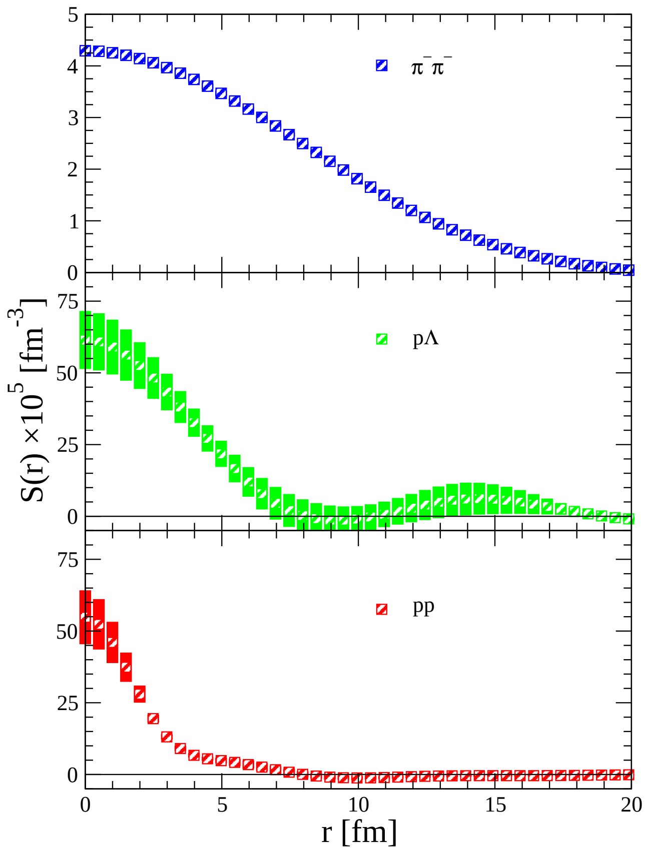
$\pi^-\pi^-$, pΛ, and pp correlations in Au+Au at 6 GeV/A ($b < 7$ fm).



- None of data Coulomb corrected (even the π's !)
- Used Reid '93 Soft-core pot. for pp (PRC 49: 2950 (1994))
- Used phenomenological pot. of Bodmer & Usmani for pΛ (NPA 477: 621-651 (1988))

Data and analysis from P. Chung *et al.*, in preparation.

AGS-E895 SOURCES



Fit sources to Gaussian:

$$S(r) = \frac{\lambda}{(2\sqrt{\pi}R)^3} \exp\left(-\frac{r^2}{4R^2}\right)$$

	R [fm]	λ
$\pi^-\pi^-$	5.39 ± 0.09	0.300 ± 0.012
$p\Lambda$	2.49 ± 0.34	0.425 ± 0.122
pp	1.22 ± 0.19	0.044 ± 0.015

- λ_{π^-} is 50% lower than result from fit to Coulomb corrected corr.
- R_{π^-} consistent with previous results
- $p\Lambda$ results need more discussion...

AGS-E895 pΛ Sources

- Assume cheezy static, spherically symmetric emission functions:

$$D_i(\mathbf{r},t) = \delta(t - t_{f/o}) D_i^0 \exp\left(-\frac{r^2}{2R_i^2}\right)$$

with $D_i^0 = f_i/(\sqrt{2\pi}R_i)^3$, $0 < f_i < 1$ is fraction of particle i

- Can show 2-particle Source function is:

$$S_{ij}(r) = \frac{f_i f_j}{\sqrt{2\pi}\sqrt{R_i^2 + R_j^2}} \exp\left(-\frac{r^2}{2(R_i^2 + R_j^2)}\right)$$

- Taking pp numbers, can figure out Λ emission ftn. parameters:

$$R_\Lambda = 3.30 \pm 0.50 \text{ and } f_\Lambda = 2.02 \pm 0.33 \leftarrow \text{ ack!}$$

Are we sampling different parts of the proton emission function?

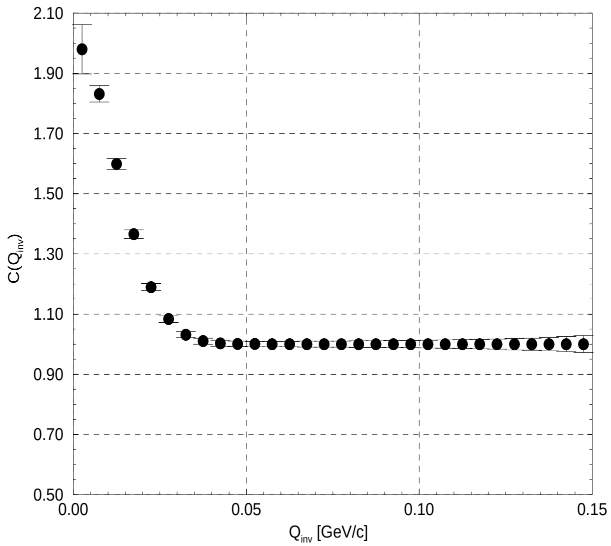
Lessons for RHIC

- Pion mystery = proton mystery?
- pΛ sensitive to different part of p emission function?
- Can compare sources from diff. particles to extract new physics.

$\triangleright$RHIC year 2 results should be interesting...

Coulomb correct π data and assume that $\Phi_{\mathbf{q}}(\mathbf{r})$ is a free wave giving:

$$|\Phi_{\mathbf{q}}(\mathbf{r})|^2 = \cos(2\mathbf{q}\cdot\mathbf{r}) + 1$$



Traditional approach:

- looks Gaussian $\Rightarrow$ fit to Gaussian
- Fourier transform of a Gaussian is a Gaussian
- Gaussian fit assumes a Gaussian source

But then again...

What about FSI?

What about non-Gaussian features?

We can do better...

Representing the 3D Source

Write the source in pair center of mass frame and expand source in $Y_{\ell m}$'s:

$$S(\mathbf{r}) = \sqrt{4\pi} \sum_{\ell=0}^{\ell_{max}} \sum_{m=-\ell}^{\ell} S_{\ell m}(r) Y_{\ell m}(\hat{r})$$

- Spherical harmonic expansion has several advantages:
  - $\ell = m = 0$ term of source can also be gotten from 1D correlation
  - Computation of kernel may be economized (6D integrals $\rightarrow$ 4D)
- Expand radial functions in Basis splines, e.g. $S_{\ell m}(r) = \sum_j S_{j\ell m} B_j(r)$

This allows us to impose constraints on source, e.g. that $\mathbf{r} = 0$ fm is a max. This implies:

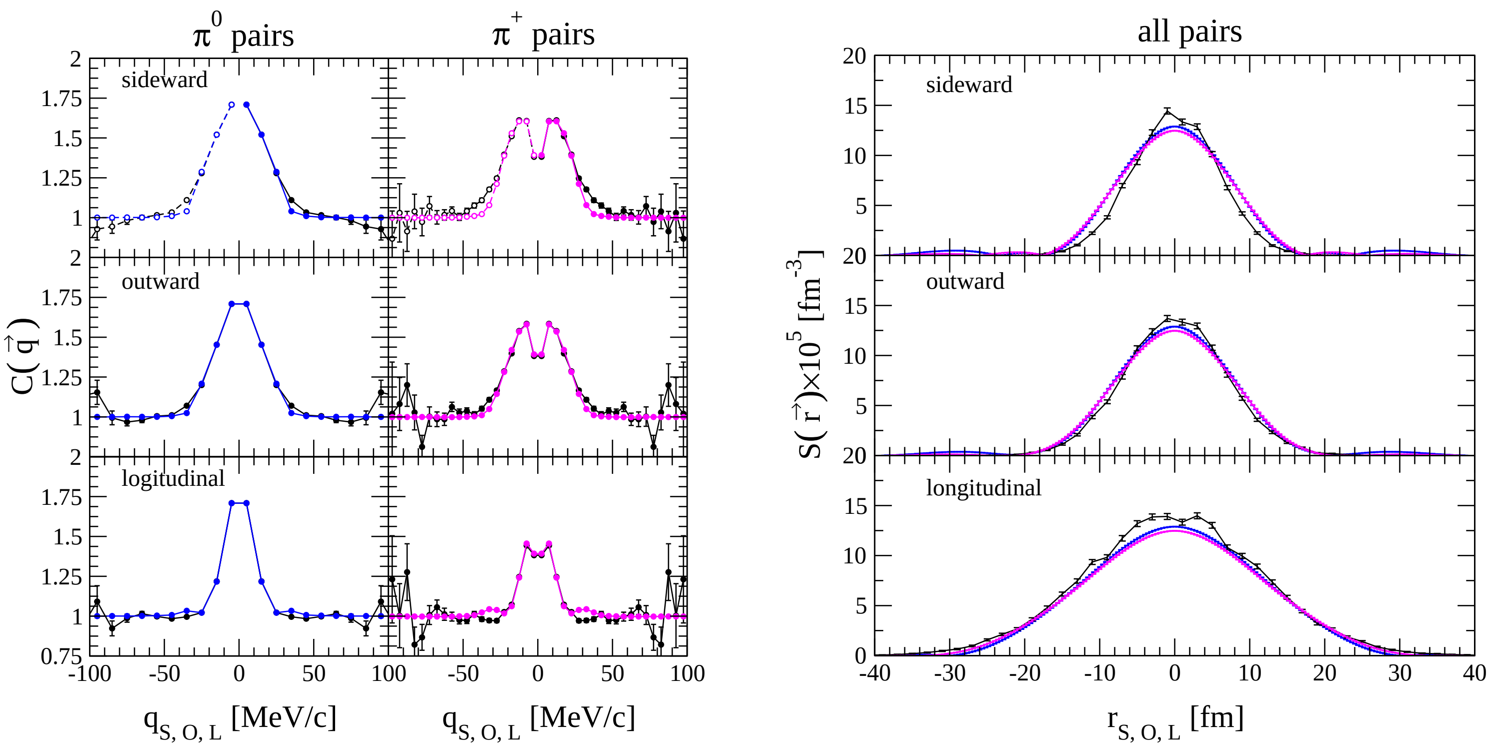
$$\frac{\partial S_{00}}{\partial r}(r \rightarrow 0) = 0 \quad \text{and} \quad S_{\ell m}(r \rightarrow 0) = 0 \text{ for } \ell, m \neq 0$$

A Simple Test

Used CRAB to generate correlation from simple single particle source:

$$D(\mathbf{r},\mathbf{p},t) \propto e^{-\mathbf{p}^2/2mT} \exp\left(-\frac{x^2+y^2}{(4\ \mathrm{fm})^2} - \frac{z^2}{(8\ \mathrm{fm})^2}\right),\ T = 10\ \mathrm{MeV}$$

We constrained source at $r = 0$ fm and at $r = r_{\mathtt{max}}$



These results are comparable in quality to Gaussian fit.